# letters to nature

## On falling through a black hole into another universe

AMONG the exact solutions to Einstein's field equations of general relativity are those describing black holes which rotate and/or carry electric charge. These solutions are novel in that they may be analytically continued through the black hole interior in a time-like direction into a succession of asymptotically flat spacetime regions that are inaccessible in the spacetime region in which the black hole first formed. There has been speculation about whether matter could travel through such black holes into these 'other universes'. As real black holes will never be precisely non-rotating and electrically neutral, it is of interest to determine whether such a transfer of matter from one universe to another is possible. If it were, it might imply the possibility of matter appearing explosively in our region of spacetime through white holes. The exact solutions of interest are suspect because they are idealised. For example, they exclude the effects of matter surrounding the hole, and quantum processes. It has been suggested[1,2] that the interior of the idealised black hole might be smashed in a more realistic model by unbounded blue shift effects associated with classical matter falling along the so-called inner horizon. In this way the space bridge would be destroyed by back reaction of the gravitational field due to the energetic matter. Here we demonstrate that quantum vacuum effects, similar in origin to the attraction energy between two electrically neutral conducting plates (Casimir effect), also cause an unbounded back-reaction which would smash the idealised interior geometry, even if no actual matter were falling into the hole. We do not speculate whether other analytically extendible spactimes exist which these quantum processes leave topologically unmodified, but demonstrate that the models which have been explicitly advanced to date cannot be taken seriously as space bridges.

The black hole solutions which contain the feature under discussion have the causal structure indicated by the conformal Penrose diagram, Fig. 1. We restrict discussion to the Reissner–Nordstrom solution (non-rotating, electrically charged black hole) because its spherical symmetry simplifies the calculation. However, the result depends on the causal, rather than geometrical structure of the spacetime, and we see no reason why the analysis would not extend to the rotating case.

The radial coordinate is denoted by $r$. The usual singularities at $r = 0$ are time-like, and this allows the spacetime to be continued through a bridge between the singularities, and out into another universe. The surface $r = r_+ = M+(M^2-e^2)^{1/2}$ is the outer horizon, which forms the boundary of the causal past of $\mathscr{I}^+$ in our Universe and is a surface of infinite red shift. Our attention centres on the other, inner horizon, $r = r_- = M-(M^2-e^2)^{1/2}$, which is a surface of infinite blue shift for ingoing null rays. The metric of the spacetime is

$$ds^2 = -\left(1-\frac{2M}{r}+\frac{e^2}{r^2}\right)dt^2+\left(1-\frac{2M}{r}+\frac{e^2}{r^2}\right)^{-1}dr^2+ + r^2(d\theta^2+\sin^2\theta\, d\phi^2) \quad (1)$$

We consider the modification of this spacetime by the presence of a massless, electrically neutral, quantum field, which in practice would be, for example, the electromagnetic or neutrino field. To a first approximation, we consider the background geometry to be fixed, and calculate the stress–energy–momentum tensor, $T_{\mu\nu}$, of the quantum field on this background. We shall show that even when there is no field energy falling into the black hole, one component of the stress tensor, evaluated in coordinates which remain smooth, still diverges along $r = r_-$, through vacuum polarisation effects.

There is still some controversy surrounding calculations of quantum field theory in curved spacetime, partly because the theory is only a semi-classical approximation. Fortunately, however, our result does not depend on detailed calculations from the theory of quantum fields, but can be extracted merely from the covariant conservation equation

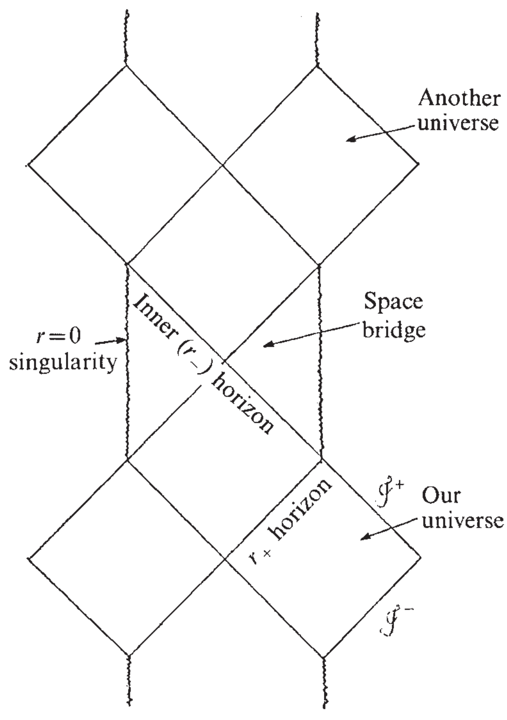
$$T^{\mu\nu}{}_{;\nu} = 0 \quad (2)$$

subject to spherical symmetry, time independence and the known form of $T_{\mu\nu}$ at spatial infinity where the spacetime is in any case flat. In particular, the only regularisation required is in the assumption of the so-called anomalous trace $T^{\mu}{}_{\mu}$. This trace has been calculated in several ways[3–9]. Moreover, in two dimensions its absence would imply that there is no particle production[3]. Thus, given the framework of this semiclassical theory we do not make any additional very strong assumptions. We follow the discussion already given by Christensen and Fulling[10] for the Schwarzschild black hole. We work in null coordinates

$$u = t-r^*,\ v = t+r^*,\ r^* = \int\frac{dr}{(1-2M/r+e^2/r^2)} \quad (3)$$

The chosen quantum state must be consistent with the assumed topology which is time-symmetric. Thus $T_{\mu\nu}$ will be a function of $r$

**Fig. 1** Conformal diagram showing the spacetime associated with an electrically charged black hole analytically extended through the black hole (space bridge) into other universes.

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only. We also require that $T_{\mu\nu}$ remain regular at the outer horizon $r = r_+$, to avoid blocking off the hole completely. Finally, we choose the quantum state to be a vacuum state of the field corresponding to no matter falling through into the hole ($T_{vv} = 0$ at $r = r_+$). Relaxing this assumption would only make the effect we predict more pronounced.

The essential qualitative features of interest are present in the two-dimensional model spacetime in which the angular coordinates in equation (1) are discarded. It turns out that the three conditions above already uniquely determine $T_{\mu\nu}$, once the anomalous trace $T_\alpha{}^\alpha$ is supplied:

$$T_\alpha{}^\alpha = \frac{R}{24\pi} = \frac{M}{6\pi r^3}\left(1-\frac{3e^2}{2Mr}\right) \tag{4}$$

A simple integration of equation (2) then yields for the ingoing null flux

$$T_{vv} = \frac{1}{24\pi}\left[\frac{M^2-e^2}{2r_+{}^4} - \frac{M}{r^3} + \frac{3(M^2+e^2)}{2r^4} - \frac{3Me^2}{r^5} + \frac{e^4}{r^6}\right] \tag{5}$$

which vanishes at $r = r_+$. At infinity $T_{vv} = T_{uu}$, so $T_{rt}$, the net radial flux, vanishes as required by the overall time symmetry. The energy density there, $T_{uu} + T_{vv}$, does not vanish. This energy is, in fact, the Hawking thermal radiation in thermodynamic equilibrium with the black hole[11]. (We could not relax the equilibrium and allow the black hole to evaporate, because it would no longer have the topology shown in Fig. 1. Whether an evaporating, charged black hole would have some other topology involving analytic continuation to another asymptotically flat spacetime region is, of course, unknown.)

To examine the physical situation represented by $T_{vv}$ at $r = r_-$, we must transform to a coordinate system that is non-singular along this null surface. One such transformation is

$$V = \tan^{-1}\left[\exp\left(\frac{r_+-r_-}{2nr_-{}^2}v\right)\right] \tag{6}$$

where $n$ is an integer $\geq 2r_+{}^2/r_-{}^2$. Thus

$$T_{VV} = \left(\frac{nr_-{}^2}{r_+-r_-}\right)^2 \frac{1}{\sin^2 2V} T_{vv} \tag{7}$$

The horizon $r = r_-$ is represented by the null surface $V = \pi/2$ in these coordinates. Clearly $T_{VV}$ will become singular at $r_-$ unless $T_{vv}$ vanishes there. Substituting $r = r_-$ into equation (5) yields

$$T_{vv}(r = r_-) = \frac{(M^2-e^2)}{48\pi}\left(\frac{1}{r_+{}^4} - \frac{1}{r_-{}^4}\right) \tag{8}$$

which is non-zero. Hence $T_{VV}$ is singular on the inner horizon. This result has also been reported by Hiscock[12] in a slightly different context.

In the more realistic four-dimensional case, using the full metric[1], it is not possible to evaluate $T_{\mu\nu}$ directly by regularisation of mode sums in terms of known functions. In two dimensions the spacetime is conformally flat, enabling simple exponential mode solutions to be used, but in four dimensions the spacetime is not conformally flat. Physically this implies that radiation can be backscattered off the spacetime curvature, which means that an outgoing energy flux reflects back into the black hole and vice versa. The effect is very complicated and manifests itself in the radial wave modes, which have to be handled numerically or approximately[10,13]. Fortunately, it is not necessary to know the exact form of $T_{\mu\nu}$ to investigate whether it remains finite at the inner horizon.

Following along the lines of the treatment for the two dimensional case, we integrate the conservation equation (2), subject to spherical symmetry, time-independence, finitude on $r_+$ and zero net flux at large $r$. The result is

$$T_{vv} = \frac{1}{2r^2}[G(r)+H(r)]+\tfrac{1}{2}\left(1-\frac{2M}{r}+\frac{e^2}{r^2}\right)[\theta(r)-\tfrac{1}{4}T_\alpha{}^\alpha(r)] \tag{9}$$

where

$$\theta(r) \equiv T_\theta{}^\theta(r)-\tfrac{1}{4}T_\alpha{}^\alpha(r) \tag{10}$$

$$H(r) \equiv \tfrac{1}{2}\int_{r_+}^{r} (\bar{r}-M)T_\alpha{}^\alpha(\bar{r})\,d\bar{r} \tag{11}$$

and

$$G(r) \equiv 2\int_{r_+}^{r} \left(\bar{r}-3M+\frac{2e^2}{\bar{r}}\right)\theta(\bar{r})\,d\bar{r} \tag{12}$$

Once again, the anomalous trace $T_\alpha{}^\alpha$ is known. In this spacetime it is

$$\begin{aligned} &(1440\pi^2)^{-1}\left[+\frac{53e^4}{r^8} - \frac{84e^2M}{r^7} + \frac{42M^2}{r^6}\right] \quad \text{for neutrinos} \\ &(360\pi^2)^{-1}\left[-\frac{47e^4}{r^8} + \frac{156e^2M}{r^7} - \frac{78M^2}{r^6}\right] \quad \text{for photons} \end{aligned} \tag{13}$$

Unlike two dimensions, $T_{vv}$ is not completely determined by the trace; there is the unknown angular component $T_\theta{}^\theta$, the evaluation of which requires the integration of the radial mode sum. $T_\theta{}^\theta$ will be a complicated function of $e$, $M$ and $r$ which cannot be written down, like the trace, in terms of simple inverse powers of $r$.

In order for the $v$–$v$ component of $T_{\mu\nu}$ to remain finite in the coordinate system equation (6), which is regular at $r = r_-$, $T_{vv}$ must approach zero as $r \to r_-$. From equation (9) this requires

$$G(r_-)+H(r_-) = 0 \tag{14}$$

$H(r_-)$ may be evaluated exactly using equations (11) and (12). The result is, for photons plus neutrinos

$$H(r_-) = \frac{1}{2880\pi^2}\left[\frac{45}{2}\frac{e^4}{r^6} - \frac{135}{7}\frac{e^4M}{r^7} - 108\frac{e^2M}{r^5} + 90\frac{e^2M^2}{r^6} + \frac{135}{2}\frac{M^2}{r^4} - 54\frac{M^3}{r^5}\right]_{r_+}^{r_-} \tag{15}$$

which is non-zero (except perhaps on a set of measure zero in $e$ space, corresponding to roots of equation (15)).

Unfortunately, it is not possible to evaluate $G(r_-)$ using known functions, because of the complexity of the radial modes which enter into $T_\theta{}^\theta$, which will be a complicated function of $e$, $M$ and $r$. However, because of equation (15), the left hand side of equation (14) is non-zero, except under the improbable circumstances that the complicated mode-dependent integral (12) exactly cancels the simple polynomial equation (15) on some open set of $e$, $M$ values. Even if this were to occur, it would not be of interest in the real universe because the functions $T_\theta{}^\theta$ and $G(r)$ depend on the choice of field modes, and hence the quantum state. It is a non-local, non-geometrical quantity. However, $T_\alpha^{\ \alpha}$, which determines equation (15), is a local, geometrical scalar, and is independent of the quantum state. Thus, any slight perturbation about the precise thermodynamic equilibrium state chosen here, which obviously will occur in the real universe, will alter $G(r_-)$ but not $H(r_-)$, and destroy the exact cancellation. Similar remarks apply to the existence of any roots of equation (15).

These general conclusions are unchanged if, instead of treating the maximally extended spacetime, we deal with a black hole formed by an imploding object. In two dimensions $T_{vv}$ cannot be affected on causality grounds (this was the case treated by Hiscock[12]). In four dimensions, backscattering of radiation emitted by the collapsing matter will occur, and this can contribute to $T_{vv}$ along $r_-$. However, it will depend sensitively on the details of the collapse, which are in general very complicated. (It is only in the region just outside $r_+$ that $T_{\mu\nu}$ is independent of the collapse details. This was the situation considered by Hawking[11].) So again, exact cancellation of $T_{vv}$, even if it should be contrived for some particular model of the collapse, would not occur in the real universe, because these different contributions which go to make up $T_{vv}$ depend on different physical circumstances for their values.

In practice, back-reaction of the quantum fields would become important near $r_-$, with the result that the interior of the Reissner–Nordstrom black hole would be smashed. Consequently, this model of a charged black hole with a space bridge to other universes is an unrealistic idealisation.

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# Hard X-ray observations of white dwarf binary systems

THE ability of degenerate dwarfs, as opposed to neutron stars, to radiate at X-ray wavelengths has created much theoretical interest. Ariel 5 hard X-ray observations indicate that current theories seem inadequate to explain all aspects of the observations. Using the Imperial College hard X-ray scintillation telescope (ST) on Ariel 5, several short period (up to 12 h) binary star systems have been studied and are described here. This study was carried out after a very hard X-ray flux was detected from AM Herculis leading to a search for similar signals from optically similar star systems. The detector used consists of 8 cm² of CsI crystal actively collimated to 8° FWHM and covers the energy range 26–1,200 keV. It is offset from the satellite spin axis, thereby enabling background subtraction using a modulation technique. The instrument and its operation are described fully elsewhere[1].

A search for hard X-ray emission from DQ Herculis was carried out during 18–22 February 1977. This white dwarf binary system has been proposed as a possible hard X-ray source by Fabian *et al.*[2] and Katz[3]. Detailed study of the signal that was detected revealed that it was inconsistent with the position of DQ Herculis but fitted AM Herculis. The fit to a source positioned at AM Herculis' position was, in fact, sufficiently good to enable upper limits on any flux from DQ Her to be set. The results are shown in Fig. 1, together with data from the Ariel 5 Sky Survey Instrument[4] and the OSO-8 proportional counter experiment[5]. The best fit to the data is a power law with exponent (−0.84). Any thermal-type spectrum is inconsistent with the observations as a whole and can only be used to explain part of the data (for example, thermal bremsstrahlung with $kT \geqslant 400$ keV could fit the high-energy tail, but still not as well as the power law). The net probability that the ST results are due to counting statistics is $1.1 \times 10^{-6}$. Also shown are the two standard deviation upper limits for DQ Herculis for three energy bins assuming a similar spectral form. The lowest energy bin comes from the Ariel 5 Sky Survey Instrument (B. Cooke, personal communication).

Due to poor time resolution in the mode of observation used (> 512 s) no search was carried out for spin periods such as those seen in the optical emission of DQ Herculis[6]. However, a search through the data for modulation with the binary periods of AM Her or DQ Her was carried out using periods published Swank *et al.*[5] and Nelson[6], respectively. It was hoped that this would help in distinguishing the sources, but no very conclusive results were obtained. Slightly stronger modulation was found with the period of AM Her, enabling a two standard deviations upper limit at 76% to be set for any modulation in the energy interval 260–1,200 keV in phase with the soft X rays[5].

A search was then made through the Ariel 5 hard X-ray data for other possible short-period binaries that may emit X rays.

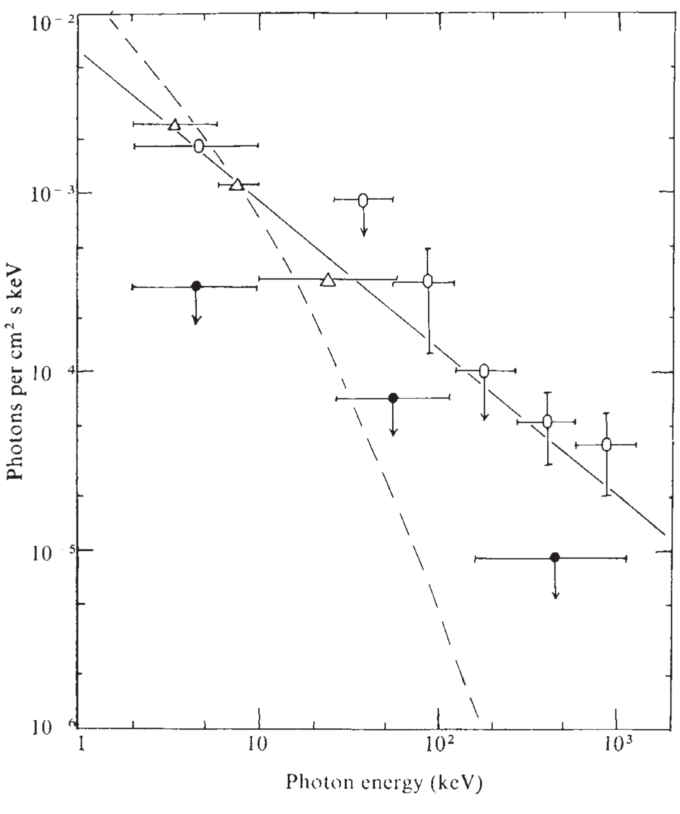


**Fig. 1** Spectral observations of AM Herculis and DQ Herculis showing (○) Ariel 5 data on AM Her; (△) OSO-8 data on AM Her; and (●) Ariel 5 two standard deviations upper limits on DQ Her. The solid line represents the power law fit to all the AM Her data, and the dashed line represents the thermal prediction of Katz[3].